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American Mathematical Association of Two-Year Colleges

The TI-86 and Some Topics from Algebra and Calculus

Presented by

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Introduction: The graphical representation of functions is an important tool for understanding calculus concepts. This graphical understanding is obtained by graphing many different functions. The graphing calculators enable us to generate these graphs with little time. The purpose of this presentation is to familiarize you with some capabilities of the TI-86 in evaluating functions and expressions, graphing functions, and some important topics in elementary calculus.

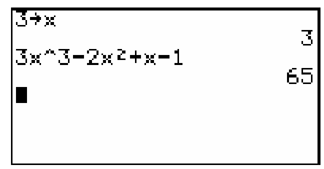
I. Evaluating algebraic expressions and functions

In order to graph a function $f(x)$ by plotting points, you need to evaluate the function for different choices of its independent variable x. There are several ways to do this task with the TI-86.

a. To evaluate an expression or a function such as $f(x) = 3x^3 - 2x^2 + x - 1$ just once, you can work in the home screen. Suppose you need to evaluate $f(x)$ at $x = 3$, do as follows.

Z 3 W 3 1 3 a

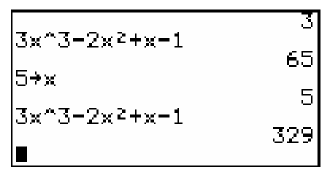
Z 3 1 3 ? 3 Z 3 S 3 Y 3 1 3 H 3 [3 1 3 S 3 X 3 a



b. If you need to evaluate the same expression for a different value of x, for example 5, recall the line that you stored x in, edit the value of x, recall the expression, and evaluate.

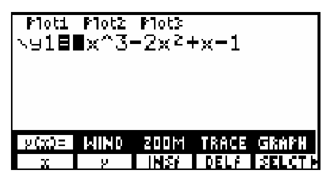
,@a : , 3 a 3 3 Qja

, p aq,ra w ax



c. To evaluate the same expression repeatedly for different values of x without recalling the expression over and over again, you can use the function edit screen to enter and save the expression as an equation and evaluate it repeatedly.

5 3 % 3 Z 3 1 3 ? 3 Z 3 S 3 Y 3 1 3 H 3 [3 1 3 S 3 X 3 - 3 -



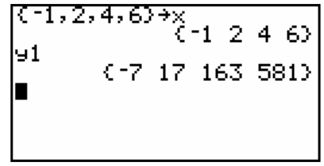
Z 3 W 3 1 3 a
, 3 v 3 . 3 (3 " 3 a 3 a

R 3 W 3 1 3 a
, 3 v 3 . 3 (3 " 3 a 3 a

d. If you need to evaluate the same expression for different values of x, for example –1, 2, 4, and 6, either repeat the process or evaluate the expression using lists. This way you store x as a list, evaluate the expression for that list and the answer will be a list that has a one to one correspondence with x.

, 3 “ 3 % 3 ` 3 X 3 0 3 Y 3 0 3 P 3 0 3 R 3 & 3 W 3 1 3 a ”

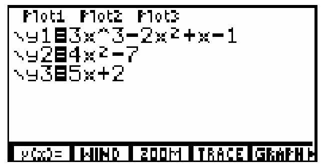
, 3 v 3 . 3 (3 " 3 a 3 a

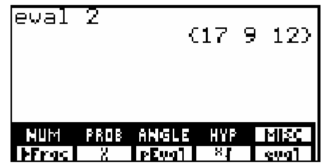


e. Evaluation of algebraic expressions and functions can also be done by TI-86 built-in functions.

1. **Eval: Eval** returns a list containing the y-value of all defined and selected functions that have already been entered in the edit function screen. Suppose y_1 is the expression in previous examples, $y_2 = 4x^2 - 7$, and $y_3 = 5x + 2$, then **Eval** 2 will return {17, 9, 12}.

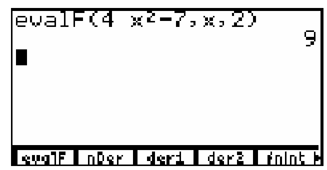
, 3 ‹ 3) 3 . 3) 3 Y 3 a



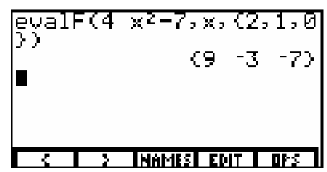


2. **EvalF: EvalF** returns the value of a given expression with respect to a variable at a given value. It also returns a list containing the values of expression evaluated with respect to the variable at each element in the list.

, 3 … 3 % 3 P 3 1 3 H 3 S 3 I 3 0 3 1 3 0 3 Y 3 D 3 a

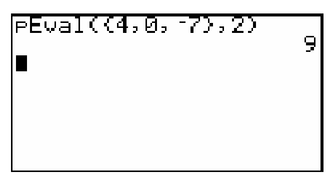


, 3 … 3 % 3 P 3 1 3 H 3 S 3 I 3 0 3 1 3 0 3 , 3 “ % 3 Y 3 0 3 X 3 0 3 ^ 3 & 3 D 3 a



3. **pEval:** Using **pEval** is the fastest and easiest way to evaluate a polynomial at a given value of its independent variable. The coefficients of the polynomial are given in a list, in order of a complete standard polynomial, and evaluated for a given value. For multiple values, repeat the process. The following example evaluates $y = 4x^2 - 7$ for $x = 2$.

, 3 ‹ 3) 3 . 3 ' 3 , 3 “ 3 % 3 P 3 0 3 ^ 3 0 3 ` 3 I & 3 0 3 Y 3 D 3 a

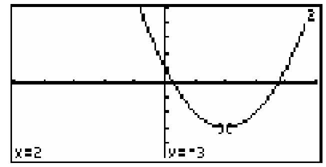


4. **EVAL in Graph Menu**: **Eval** evaluates a function for a specified x when graph is displayed on home screen. Suppose you need to evaluate $y = x^2 - 4x + 1$ at $x = 2$. After graphing y, select **EVAL** from the **GRAPH** menu. Enter 2 at **x =** prompt and enter. The coordinate values then are displayed on Graph-Screen. The only thing you need to be cautious about is

that the value of *x* has to be within the window parameters. If you have multiple graphs, use # 3 " keys to maneuver from one to another.

5 3 % 3 1 3 H 3 S 3 P 3 1 3 [3 X 3)

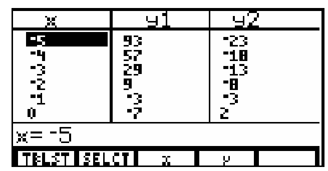
. 3 . 3 % 3 Y 3 a 3



So at $x = 2, y = -3$.

f. Multiple values of dependent variable(s) of equation(s) entered, using edit function menu, can be viewed using **TABLE** feature of TI-86. **TABLE** will display the values of all existing equations in edit function menu for an arithmetic sequence of the independent variable. Suppose in edit function menu we have the following equations, $y_1 = 4x^2 - 7$ and $y_2 = 5x + 2$. 6 3 & will activate the **TABLE** set-up screen. Choose a starting value for *x* (**TblStart**) and an increment (**ΔTbl**). Leave the **Indpnt** option on **AUTO**. **TABLE** will display values of *x* starting at **Tblstart** with steps of **ΔTbl**. The **Ask** feature of **Indpnt** option can be chosen if non-sequential values of *x* is desired.

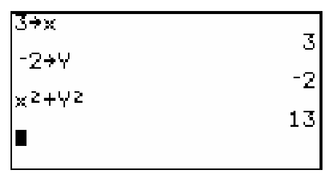
6 3 & 3 %



g. To evaluate an expression with more than one variable, you can use the home screen to do the job. Suppose you need to evaluate $Z = x^2 + y^2$ for $x = 3$ and $y = -2$. Evaluation in home screen is as follows.

Z 3 W 3 1 3 a
` 3 Y 3 W 3 0 3 Y 3 a

1 3 H 3 [3 0 3 Y 3 H 3 a 3



The other option is to write a program to do a multiple calculation without having to enter the expression over and over again. The following program will calculate the monthly payment of a purchase or a loan.

```
:ClLCD
:Disp " "
:Disp "   This program"
:Disp "    calculates"
:Disp "  monthly payments"
:Disp "   of a loan or"
:Disp "   a purchase"
:Disp " "
:Disp "            [ENTER]"
:Pause
```

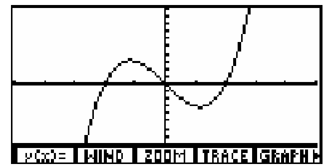
```
:ClLCD
:Disp "  Enter the amount"
:Disp "     of loan or"
:Disp "      purchase"
:Input p
:ClLCD
:Disp "Enter number of years"
:Disp " "
:Input t  :nn=t*12
:ClLCD
:Disp "Enter interest rate"
:Disp " "
:Input r
:i=r/1200
:K=i/(1-(1+i)^(-nn))
:pay=p*K
:ClLCD
:Disp "    Your monthly"
:Disp "    payments are"
:Disp round(pay,2)
:Stop
```

SOLVER can also be used to evaluate expressions with multiple variables. We look at **SOLVER** later.

II. Graphing functions.

In order to graph function(s), you need to enter the function(s) using edit function screen. Then, using the graph command you graph the function(s). If in edit function screen you have multiple entries and you just need to graph one or a few of the functions, use select command to select the desired ones. Use appropriate graph menu items such as **[WIND] [ZOOM] [TRACE]** to see the best view of the graph. In the following example we graph $y = x^3 - 4x$.

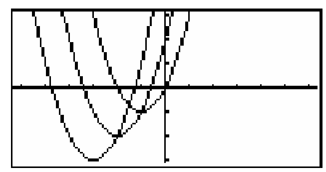
5 3 % 3 1 3 ? 3 Z 3 S 3 P 3 1 3 - 3
)



You can also graph a family of curves, draw horizontal and vertical lines, line segments, circles, and inverse of a function.

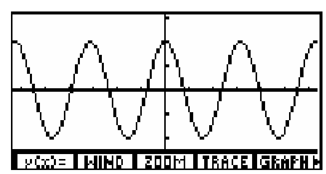
a. **A family of curves:** Suppose you need to demonstrate various shifts of a parabola $y = a(x-h)^2 + k$ for $h = -1, -2, -3$ and $k = -1, -2, -3$. $-1, -2,$ and -3 will be entered as lists in edit function screen. The following key strokes will graph $y = (x+1)^2 - 1$, $y = (x+2)^2 - 2$, and $y = (x+3)^2 - 3$. Adjust the window to see all graphs.

5 3 % 3 C 3 % 3 [3 , 3 “ 3 % 3 X 3 0
3 Y 3 0 3 Z 3 & 3 DH 3 S 3 % 3 S 3 X
3 0 3 S 3 3 Y 3 0 3 S 3 3 Z 3 & 3 - 3
)



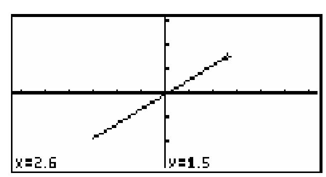
b. **Drawing a function from a Home-Screen.** A function can be graphed directly from home-screen using **DrawF** from **Graph Draw** menu or entering **DrawF** and function in home-screen.

5 3 . 3 & 3 . 3 % 3 Y = C Y 1 D 3 a
3 3



c. **Drawing horizontal and vertical lines, line segments, and circles:** For drawing a line segment on your graph, select **LINE** from **Graph** menu. Choose the segment ends (coordinates) by using cursor. **Draw** the line.

5 3 . 3 & 3 & 3 # 3 # 3 a 3 " 3 " 3 a
3 3

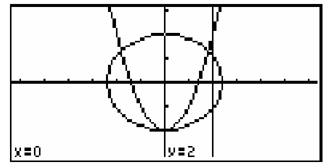


By choosing **VERT**, **HORIZ**, or **CIRCL** on graph draw menu you can draw a vertical line, horizontal line, and circle.

5 3 . 3 & 3 (3 # 3 # 3 a 3 " 3 " 3 a

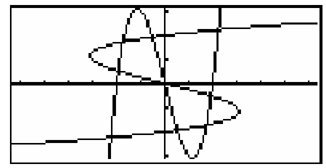
5 3 . 3 & 3 ' 3 ! 3 ! 3 a 3 3 3 a

5 3 . 3 & 3) 3 # 3 # 3 a 3 " 3 " 3 a



d. **Graphing a function and its inverse:** Suppose you need to graph the inverse of $y = x^3 - 4x$. Using function edit screen, enter the function. Select **DrInv** from **GRAPH-DRAW** menu and enter. You can also type **DrInv** $x^3 - 4x$ in Home-screen to do the task.

5 3 . 3 & 3 . 3 . 3 . 3 ' 3 , 3 v
3 . (3 " 3 " 3 a 3 a



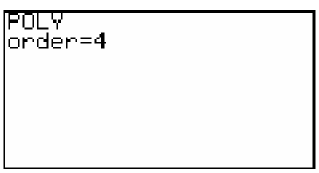
III. Further Topics in Graphing Functions.

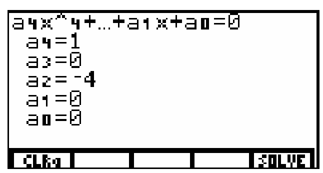
One of the most important methods of graphing complicated functions without the advantage of calculus is to find as much possible information as we can by finding critical points of the graph such as x-intercepts (roots), y-intercept, points of discontinuity, and maximum and minimum points. These critical points can always be found by using trace feature (**TRACE**) of the TI-86, but they will not always be exact.

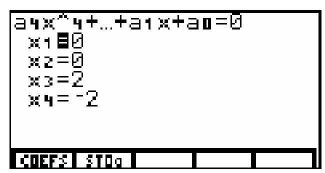
Suppose we need to graph $f(x) = [x^2(x^2 - 4)]/(x-1)$. The x-intercepts are solutions (roots) of the numerator and vertical asymptote(s) are root(s) of the denominator. In edit function screen enter $y1 = x^4 - 4x^2$, $y2 = x - 1$, and $y3 = y1/y2$. Solutions of $y1$ and $y2$ can be found using **SOLVER**, **POLY**, and **ROOTS** features of TI-86. The y-intercept, maximum, and minimum of a function, if they exist, can also be explored using TI-86.

a. **POLY** can solve only polynomial functions and will display all roots (solutions) on home-screen, or store them in a variable. In **POLY**, we enter the degree (order) of polynomial as well as the coefficient of each descending power of independent variable. Solutions of $y1$ are as follows.

, 3 u 3 P 3 a 3 X 3 " 3 ^ 3 " 3 ` 3 P 3 " 3 ^ 3 " 3 ^ 3 a 3) 3 (solve)

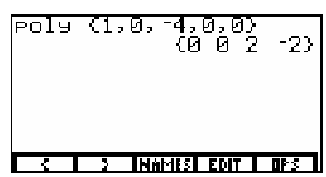






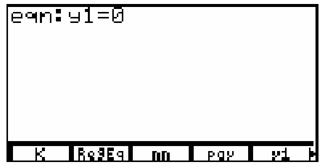
POLY can also be triggered in Home-Screen using **CATLG-VARS** followed by the coefficients of the terms of the polynomial. Calculator will display the solutions in a list. If some roots are complex the list contains n ordered pairs representing the roots in the form of $a + bi$.

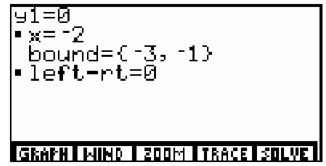
, 3 v 3 % 3 0 3 % 3 % 3 " 3 " 3 " 3 a 3 , 3 " % 3 X 3 0 3 ^ 3 0 3 ` 3 P 3 0 3 ^ 3 0 3 ^ 3 & 3 a : 3



b. **SOLVER**: Contrary to the **POLY,** which is restricted to polynomials of one variable, **SOLVER** can solve many types of equations, including multivariable equations. [2nd] **SOLVER** will start the process with **eqn** prompt on Home-Screen. The functions can be entered using keyboard or one can be chosen from the menu (if it already exists.) **SOLVER** can solve for a variable, in multivariable functions, given initial values for all but one. If after choosing a function "=" is not entered after the function, calculator automatically converts the function into an expression. **SOLVER** needs lower/upper bounds to search for solutions.

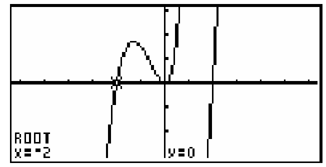
3 , 3 s 3) 3 ^ 3 a 3)
3





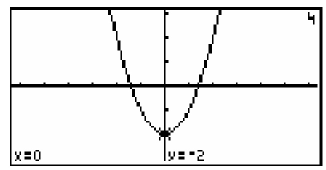
c. **ROOTS**: In **MATH** submenu of **GRAPH** menu, **ROOTS** can solve an equation given left/right bounds. **ROOTS** can be applied to any function.

5 3 . 3 ‹ 3 % 3 3 3 a 3 ! 3 ! 3 a 3
a 3 3



d. **y-intercept**: y-intercept of a function can be obtained using **YICPT** from **MATH** submenu of **GRAPH** menu. **YICPT** does not need any left/right boundaries.

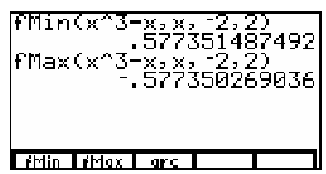
5 3 . 3 ‹ 3 . 3 &



e. **Maximum and Minimum value of a function**. Maximum/minimum of a function (local or absolute) can be found using **FMIN/FMAX** from **MATH** submenu of **GRAPH** menu. Using these functions requires lower/upper bounds.

fMin/fMax from **CALC** menu will also do the job but from Home-Screen. The expression, the independent variable, and lower/upper bounds have to be entered.

, 3 … 3 . 3 %/ & 3 a

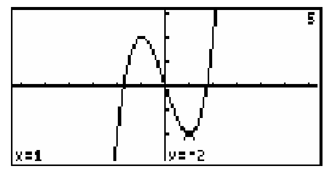


IV. CALCULUS

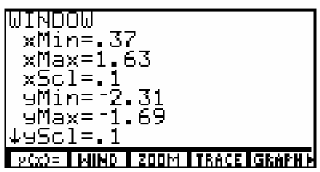
a. Limits

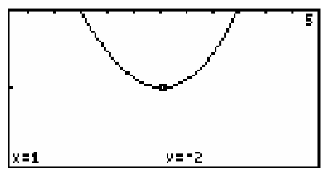
1. **Graphical approach:** Let $f(x) = x^3 - 3x$. Consider $\lim_{x \to 1} f(x)$. Since $f(x)$ is a polynomial and is continuous, then $\lim_{x \to 1} f(x)$ can be easily evaluated. If we graph $f(x)$ on an interval containing $x = 1$ and use **TRACE**, $\lim_{x \to 1} f(x)$ can be verified (set **ZOOM** to **ZDECM**.) Using **TRACE** enables us to obtain a table of $(x, f(x))$ to show that as $x \to a$, $f(x) \to L$ from left and right.

x	0.7	0.8	0.9	1	1.1	1.2	1.3
$f(x)$	-1.757	-1.888	-1.971	-2	-1.969	-1.872	-1.703

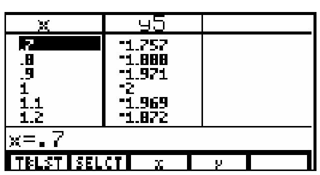


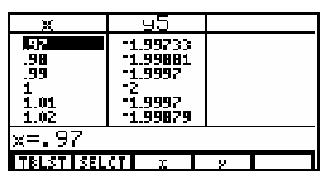
Change the window as follows and repeat the process to obtain the second table of values.



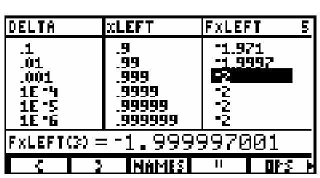


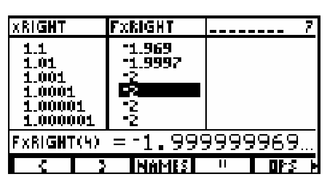
2. **Using TABLE:** In **TABLE**, with changing **ΔTbl**, we can get as close as possible to $x = 1$. First, set **TblStart** =0.7 and **ΔTbl** =0.1 and observe $f(x)$ as $x \to 1$ from left and right. In the second trial set **TblStart** =0.97 and **ΔTbl** =0.01. **Ask** option of **TABLE** can be activated to enter non-sequential values for x.





3. **Using LIST and sequence function.** Activate the list by **, 3 “** . Create five lists called, **DELTA**, **xLEFT**, **FxLEFT**, **xRIGHT**, and **FxRIGHT**. Move the cursor to the title bar of **DELTA** and enter **seq**(10^-x, x, 1, 5) to generate the list 0.1, 0.001, …, 0.000001. Enter the formula **1-DELTA** for **xLEFT** and **1+ DELTA** for **xRIGHT**. Enter **xLEFT^3-3*xLEFT** for **FxLEFT** and **xRIGHT^3-3* xRIGHT** for **FxRIGHT**. **xLEFT** and **xRIGHT** are x-values that are approaching 1 from left and right, respectively. **3** ”





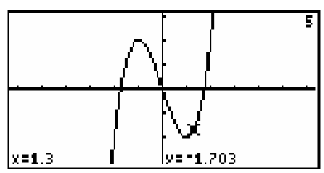
b. Numerical Differentiation:

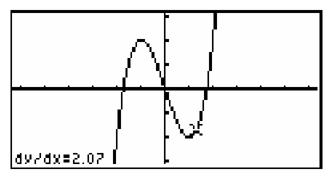
1. **Using definition:** We can describe the derivative of $f(x)$, at a point *a*, as the slope of the tangent line to the graph of $f(x)$ at the point $(a, f(a))$. This leads to the definition for the derivative as $f'(a) = \lim_{h \to 0} (f(a+h) - f(a))/h$. This limit can be dealt with as in the previous section.

2. **Central difference approximation:** As $h \to 0$ from left and right, $(f(a+h) - f(a))/h$ are the slopes of the secant lines that pass through $(a, f(a))$ on both sides of *a*. Specifically, $m_f = (f(a+h) - f(a))/h$ is called the ***forward difference approximation*** and $m_b = (f(a) - f(a-h))/h$ is called the ***backward difference approximation***. We can then approximate derivative of $f(x)$ at the point $(a, f(a))$ by taking the average of these two approximations. That is, $m_c = (m_f + m_b)/2 = (f(a+h) - f(a-h))/2h$. These formulas can be entered as functions in edit function screen, a **TABLE** or a **LIST** for numerical calculations.

3. **TI-86 intrinsic functions:** dy/dx, **nDer**, **der1**, and **der2** are TI-86's intrinsic functions for calculation of derivatives (1st or 2nd) at a point. dy/dx is under **MATH** submenu of **GRAPH** menu. Activation of dy/dx activates the trace on the Graph-Screen. Move the cursor to the desired *x*-values and then press ENTER. $f'(x)$ appears on Graph-Screen. In this example $f'(1.3)$ is evaluated where $f(x) = x^3 - 3x$.

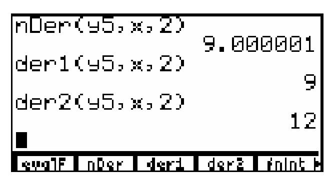
5 3 . 3 % 3 & 3 ! ! 3 a





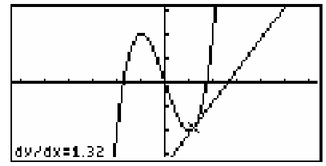
nDer, **der1**, and **der2** are under **CALC** menu. Execution of these three functions will paste them in Home-Screen ready for input. The syntax for all is (*function, variable, value*). The functions can be either directly entered on Home-Screen or a function can be chosen from **equ** lists under **CATL-VARS**.

, … & , v . (3 *y*5 **a 0 1 0 Y D**
a repeat the process for the next two.



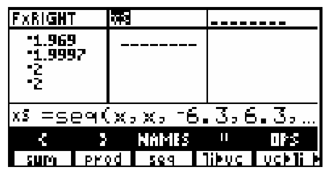
4. **The Tangent line: TANLN** in the **MATH** submenu of **GRAPH** menu will graph the tangent line at the prompt.

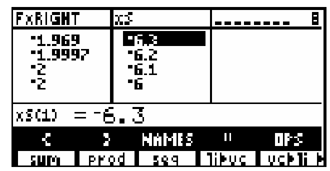
5 3 . 3 ‹ 3 . 3 . 3 % 3 ! 3 ! 3 a



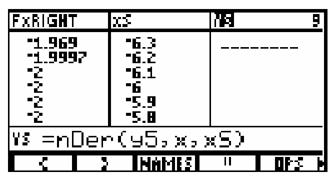
5. **Graphing the derivative of a function:** Graph of the derivative of a function can be obtained either by finding the derivative analytically and graphing that function or by plotting the slope of the tangent line for all values of x in the graph window. Suppose we need to graph derivative of $f(x) = x^3 - 3x$. Create a list, say xS. Enter the values of x from –6.3 to 6.3, with interval 0.1,for xS, by entering sequence formula at the $xS =$ prompt. Create another list, say yS. At $yS =$ Prompt enter **nDer** (y5, x, xS). The last formula generates the slope of the tangent line to the $f(x)$ at all points in xS. From **STAT** menu, plot xS versus yS.

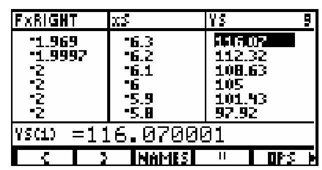
, 3 “ 3 (3 NAME=xS **a**
) 3 . 3 ' 3 x, x, -6.3, 6.3, .1
a



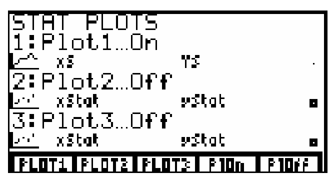


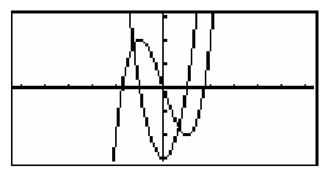
 # 3 ! 3 NAME=yS **a , 3 … 3**
& 3 y5, x, xS **a**





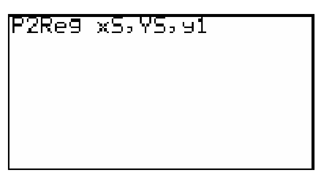
, 3 ™ 3 ' 3 % 3 a
5 3)

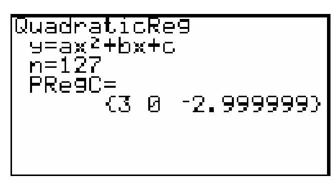




We can even go further and find the analytical derivative of $f(x)$ using the data generated in the previous section and the regression tools of TI-86. TI-86 does a number of regression analyses. All regression tools of TI-86 are under **CALC** submenu of **STAT** menu.

, 3 ™ 3 % [CALC] **3 . 3 (**
[P2REG]





Therefore, the derivative of $f(x) = x^3 - 3x$ is $f'(x) = 3x^2 - 3$.