Two Experiments with CBL (Calculator Based Laboratory) System For Mathematics and Science Classes

Exploration of Rational, Sinusoidal, Exponential, and Logarithmic Functions

A Professional Development Activity by Habib Far, Math Instructor

The CBL:

CBL (Calculator-Based Laboratory System) is a portable data collection tool for a variety of physical quantities including, temperature, motion, force, light, and sound by appropriate probes.

The Set up:

1. Connect the CBL unit to the TI-86.
2. Connect the microphone to the Channel 1 (CH 1).
3. Turn on the CBL unit and the TI-86. CBL is now ready to receive commands from the calculator.

Experiment One: Frequency, Period, and Wavelength of a Sound Wave Produced by a Tuning Fork

Introduction:

The frequency of a wave is the number of complete waves that pass a given point per unit of time. Each music note has a specific frequency measured in Hertz (1 Hz = 1 cycle-per-second). Using a tuning fork, we will discover the relationship between the frequency (*f*) of a note, the period, in seconds (*T*), of its pressure wave, and its wavelength, *lambda* (λ). The relationships are $f = 1/T$ and $\lambda = 343/f$, respectively (speed of sound is 343 m/s at $20^{\circ}C$.) In this experiment we measure the period *T* of a known tuning fork (A = 440), its frequency, and the wavelength λ of its pressure wave.

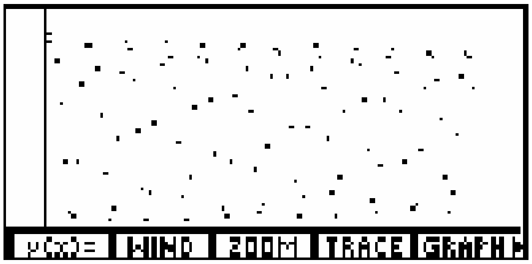
Instructions:

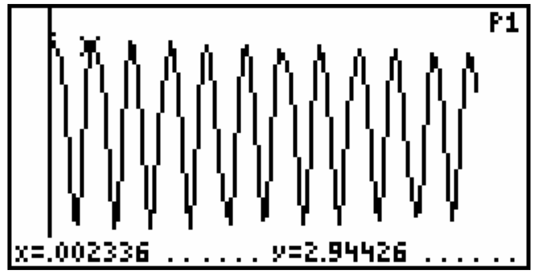
1. Turn on the TI-86 and the CBL. Start the TUNE program on calculator by pressing 7 % **[TUNE]**. The program will pause execution and wait for you to press a .

2. Strike the tuning fork and place the vibrating fork as close as possible to the microphone and a . (Don't let the fork actually touch the microphone.)

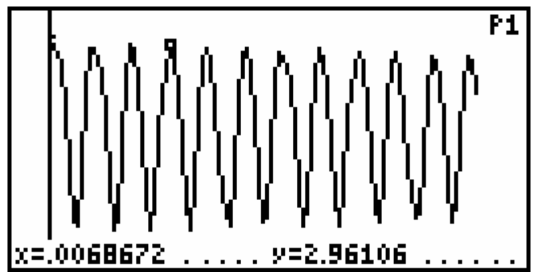
3. Pressure is stored in L_5 and time (in seconds) in L_2. Plot of pressure versus time will appear on calculator screen as CBL is collecting the data. CBL is measuring the pressure every 1/4000th of a second.

4. Change the plot to continuos graph ($x\,y$ line) by , 3 ™ 3 ' 3 % 3 " 3 & 3 **(xyLINE)** 5 5. Press ((TRACE) ! on the calculator and record the x-coordinates of first maximum (x_0) and 10th maximum (x_{10}).





In this trial $x_0 =$ 0.002336 and $x_2 =$ 0.0068672 then $x_2 - x_0 =$0.0045312. Therefore the time (period) for 2 complete waves is 0.0045312 of a second or 0.0022656 of a second per period, which gives the frequency, equal 441.4 Hz using $f = 1/T$.



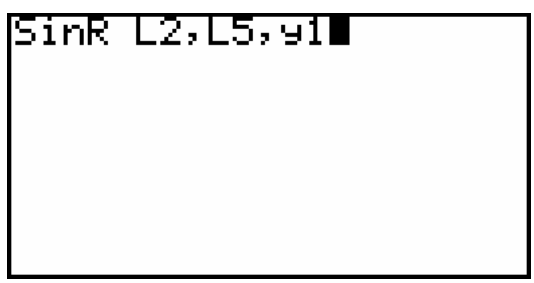
Analysis:

Since measured frequency is 441 Hz and the period $T = 0.0022656\,s$ then the wavelength λ of the pressure wave produced by the tuning fork is

$$\lambda = 343/441 = .7777^{m} = 77.77^{cm}$$

Using TI-86, sine regression can be performed to find the sine function associated with the waves produced by the tuning fork.

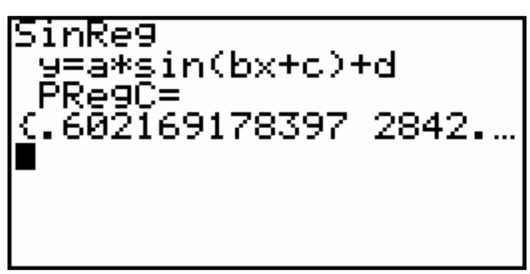
, ™ % (**CALC**) . & (**SinR**) 0 **L** Y
0 0 **L** Q 0 , 0 **y** X ¡ .



The resulting equation will be stored in y1 and simultaneously appear on Home-Screen. $

If $y = a\sin(bx + c) = d$ is general form of a sine function, then for our experiment

$$y = 0.6\sin(2842x - 0.9) + 2.6$$



Experiment Two: Superposition, Standing Waves, and Harmonic Waves

I. Frequency, Period, and Wavelength of a Sound Wave Produced by a closed-end Tube.

Introduction:

An important aspect of waves such as waves on strings, sound waves, surface water waves, and electromagnetic waves, is the combined effects of two or more traveling waves. The superposition principle states that the actual displacement of any part of a disturbed medium is equal to the algebraic sum of displacements caused by the individual waves. If the harmonic (sinusoidal) waves that combined in a given medium have the same frequency and wavelength, a stationary pattern, called standing waves, can be produced at certain frequencies and under certain circumstances.

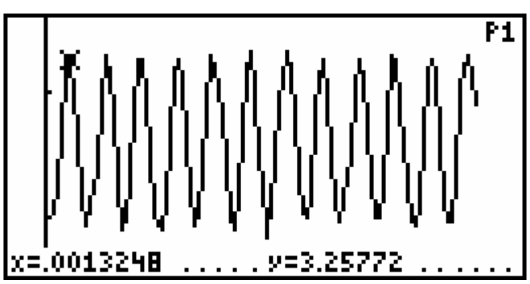
Musical instruments, such as flute and organ, make use of the natural frequencies of sound waves in hollow pipes. These frequencies depend upon the length of the pipe, its shape, and upon whether one end is open or closed.

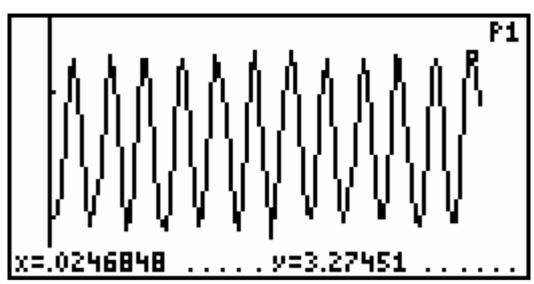
Standing waves can be set up in a pipe (tube) with a closed end. The wavelength of the fundamental standing waves, f_1, is four times the length of the tube, L. That is, $\lambda_1 = 4L$. Since $f = 343/\lambda$ then $f_1 = 343/4L$. In a tube, closed at one end, only odd harmonics, the odd multiple of f_1, are present, and they are given by

$$f_n = \frac{343 \cdot n}{4 \cdot L} \text{ for } n = 1, 3, 5, \cdots . \qquad (1)$$

Instructions:

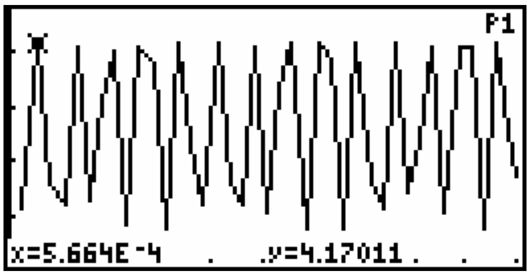
Instructions and set-ups are as the first experiment, but this time instead of a tuning fork, a piece of pipe of length 18.2 centimeters, with one closed end, is being used to measure the frequencies and wavelengths of the fundamental wave and the third harmonics produced in the pipe. We blow air into the pipe until it starts to resonate and then we start to gather the data with CBL.

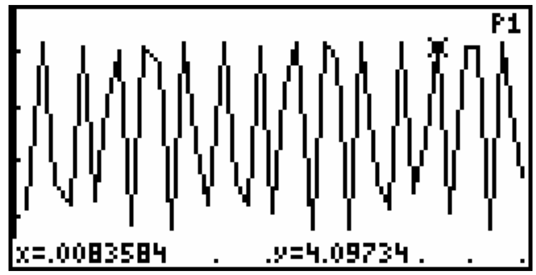




One period of the fundamental wave for the pipe is $T_1 = \frac{0.0246848 - 0.00183248}{11} =$ 0.02123636 second. Since $f = 1/T$ then the frequency of the fundamental wave $f_1 = 470.89$ Hz. Direct calculation results in $f_1 = \frac{343*100}{4*18.2} = 471.15$ Hz. The wavelength of the first harmonic is $\lambda_1 = 4*18.2 = 72.8^{cm}$.

We repeat the experiment but this time by putting more pressure on blowing air in the pipe. As the figure below, one period of the third harmonics is $T_3 = \frac{.0083584-.0005664}{11} = 0.000708364$ s. That





results in $f_3 = 1411.7$ Hz. Direct calculation results in $f_3 = \frac{343*100*3}{4*18.2} = 1413.46$ Hz. The wavelength of the third harmonics is $\lambda_3 = (4/3)*18.2 = 24.26^{cm}$.

II. Relationship between Frequency of a Musical Note and its Corresponding Location on a Piano Keyboard. A logarithmic and exponential function)

The frequency of any musical note can be found by the equation

$$f(n) = 440*2^{n/12}, \qquad (2)$$

where *n* represents the number of black and white keys below and above A_{440}. Or by the

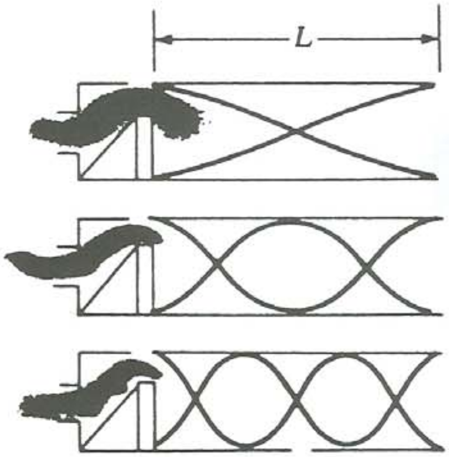
$$f(n) = (27.5)*2^{(n-1)/12}, \qquad (3)$$

where *n* represents the number of black and white keys from the left side of piano.

In our experiment we discovered that the frequency of fundamental wave for a closed-end pipe is $f_1 = 470.89$. Solving (1) for *n* using logarithm will result in

$$n = \frac{12}{\log 2} * \log\left(\frac{f}{440}\right).$$

Substituting 470.89 for *f* will results in $n \approx 1$. So our pipe is producing a $A^{\#}$ note right above A_{440}. The Third harmonics frequency results in $n \approx 20$. The 20th key above A_{440} is an F.

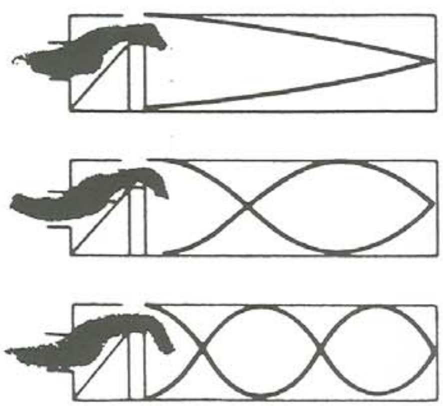


$\lambda_1 = 2L$
$f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L}$ — First harmonic

$\lambda_2 = L$
$f_2 = \dfrac{v}{L} = 2f_1$ — Second harmonic (First overtone)

$\lambda_3 = \dfrac{2}{3}L$
$f_3 = \dfrac{3v}{2L} = 3f_1$ — Third harmonic (Second overtone)

(a)

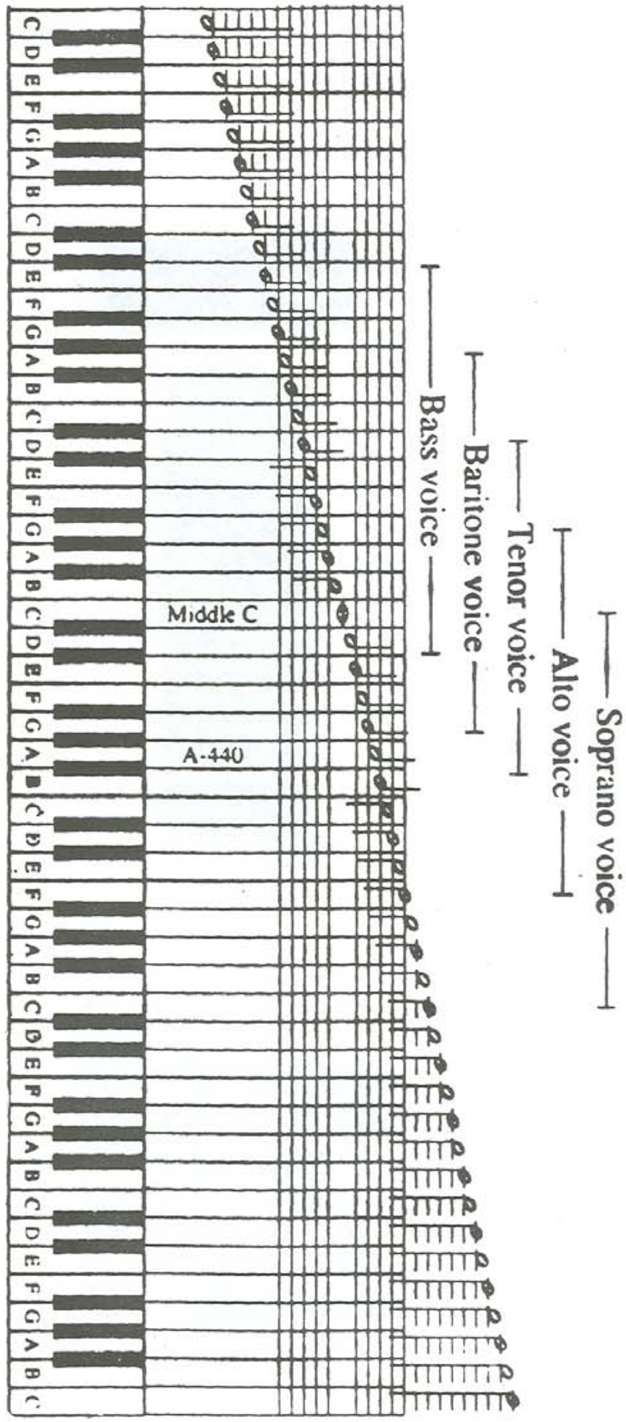


$\lambda_1 = 4L$
$f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{4L}$ — First harmonic

$\lambda_3 = \dfrac{4}{3}L$
$f_3 = \dfrac{3v}{4L} = 3f_1$ — Third harmonic (First overtone)

$\lambda_5 = \dfrac{4}{5}L$
$f_5 = \dfrac{5v}{4L} = 5f_1$ — Fifth harmonic (Second overtone)

(b)


Soprano voice
Alto voice
Tenor voice
Baritone voice
Bass voice
Middle C
A-440

(1) Physics for Scientists and Engineers, 2nd Ed., Raymond A. Serway, Saunders College Publishing, New York.
(2) College Algebra, Concepts and Models, 2nd Ed., Larson, Hostetler, Hodgkins, D.C. Heath and Co., Lexington.
(3) College Algebra, 3rd Ed., Aufmann, Barker, Nation, Houghton Mifflin Co., Boston.